

Parallel Fractal Image Generation

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What are Fractal Images?

- ◆ sets of points generated by iterating relatively simple calculations
- ◆ contain infinite detail
- ◆ show self-similarity
- ◆ have a fractal dimension

What are Fractal Images? The complex plane

What are Fractal Images? The iteration formula

$C = -1.0 + 0.5i$

n	Re z _n	Im z _n
1	-1.00000	0.50000
2	-0.25000	-0.50000
3	-1.18750	0.75000
4	-0.15234	-1.28125
5	-2.61839	0.89038
6	5.06220	-4.16279
7	7.30768	-41.65264
8	-1682.61511	-608.28115
9	2461186.65194	2047006.620882
10	1867203633030.900000	10076130739563.000000

$Z_0 = 0$

What are Fractal Images? Iteration approaching infinity

$C = -1.0 + 0.5i$

n	z _n
1	1.11803
2	0.55902
3	1.40451
4	1.29028
5	2.76584
6	6.55472
7	42.28971
8	1789.18964
9	3201399.12507
10	1024767898363.00000

$Z_0 = 0$

What are Fractal Images? Iteration converging

$C = 0.25 - 0.25i$

n	z _n
1	0.35355339059327
2	0.44068390943100
3	0.47005027989035
4	0.48997608405836
5	0.33506252161220
6	0.34958508021450
7	0.33934473986338
8	0.40606669062459
9	0.33385730589694
10	0.37488377936363
...	...
110	0.38268343236512
111	0.38268343236508
112	0.38268343236507
113	0.38268343236508
114	0.38268343236510
115	0.38268343236510
116	0.38268343236509
117	0.38268343236509
118	0.38268343236509
119	0.38268343236509
120	0.38268343236508

$Z_0 = 0$

What are Fractal Images? Iteration looping

n	Z _n
1	0.70710678118655
2	0.50000000000000
3	0.55901699437495
4	0.73154374443998
5	0.18023862492280
6	0.69951500659028
7	0.45627830615575
8	0.56915055236267
9	0.68760238111038
10	0.24992959329357
...	...
140	0.48608739228658
141	0.48154429128127
142	0.52267815373996
143	0.45621548679314
144	0.52219422862102
145	0.48025876790393
146	0.4882527912022
147	0.51699488737534
148	0.45968041794762
149	0.52254879060228
150	0.47865161324317

$Z_0 = 0$
 $Z_{n+1} = Z_n^2 + C$
 $C = -0.5 - 0.5i$

What are Fractal Images? The Mandelbrot set

$Z_0 = 0$
 $Z_{n+1} = Z_n^2 + C$

What are Fractal Images? Image characteristics

Iteration maximum: 150

- ◆ self-similarity
- ◆ infite detail

What are Fractal Images? Image characteristics

Iteration maximum: 1,500,000

- ◆ more iterations
- higher accuracy

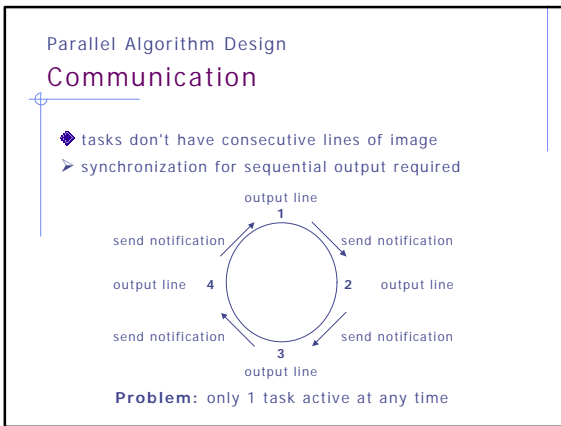
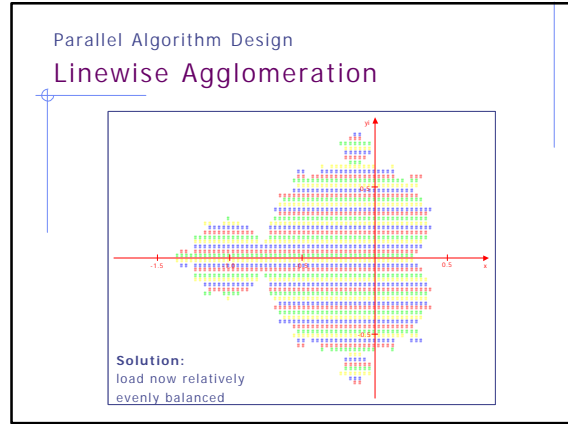
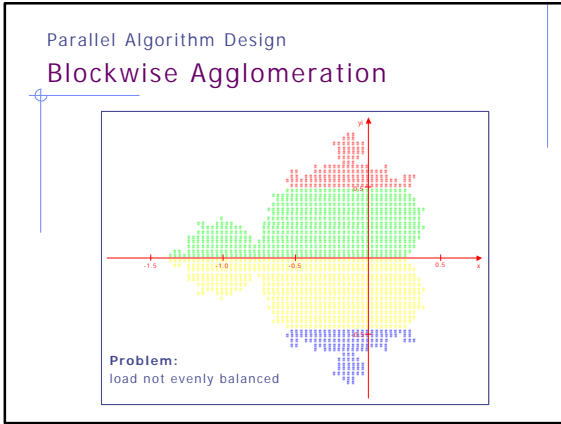
Parallel Algorithm Design

- ◆ Motivation:
 - deep zooms require high iteration count
 - max. iteration count always reached on the inside of the set
 - high image resolutions desired
 - e.g. 8x11" fractal at 600 dpi: 31,680,000 pixels, each with <= 2,000,000 iterations
- long computation times with serial algorithms
- ◆ parallel computing can decrease computation time

Parallel Algorithm Design Partitioning

smallest unit of computation: 1 pixel

ASCII output:
= Z approaches infinity
. = Z does not approach infinity



- Parallel Algorithm Design
Non-blocking communication
- Solution: use non-blocking communication
 - while lines to compute
 - non-blocking receive from previous node
 - compute lines and store in output buffer
 - if notification received
 - if at least one line in buffer
 - send first line and delete from buffer
 - else
 - continue computation till one line complete
 - send line and delete from buffer
 - non-blocking send notification to next node

